**A Generalized Prospective Modelling Framework**

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## 1.0 Introduction

Recent PATH analyses, and reviews of these analyses, have resulted in several generalized linear Ricker stock-recruit models that could be used in prospective/decision analyses of management actions for the survival and recovery of Snake River spring/summer chinook. The purpose of the model used in a decision analysis is to forecast the outcomes (expressed as pre-defined performance measures) of the alternative management actions under consideration, while incorporating multiple hypotheses about key uncertainties. To accomplish this, the prospective model should conform to the following set of principles:

1. It should be relatively simple to implement alternative hypotheses.
2. The model should allow for a common method of expressing alternative hypotheses.
3. The implications of alternative hypotheses on the decision to be made should be clear.
4. The model should be easy to explain in non-technical terms to decision makers.

Although multiple models could satisfy some of these principles, it would be preferable to use a single modelling framework that was flexible enough to incorporate all “plausible” uncertainties and hypotheses. There are several advantages of using a single modelling framework: First, it would greatly facilitate principles 2 and 3. Second, it would be easier to explain differences and similarities between alternative hypotheses using a common modelling framework. Third, it would avoid time-consuming parallel model runs and comparisons of model output, where it is difficult to separate the effects of different model structures from the effects of different hypotheses. Previous efforts to compare passage models experienced these difficulties (ANCOOR 1993, 1995)

Discussions at a June 26-27 meeting of the prospective modelling workgroup and subsequent conference calls have resulted in a common, generalized prospective modelling framework, with several alternative hypotheses about some of its key components. Complete descriptions and justifications for these hypotheses are provided in other papers in this package of PATH products for SRP review:
“Prospective analysis for the alpha model” (Anderson and Hinrichsen 1997)
“Draft proposed general framework for prospective modeling with detailed examples for one hypothesis about delayed mortality” (Wilson et al. 1997)

The purpose of this document is to provide some context for these papers by:

- Summarizing the potential prospective models (Section 2)
- Commenting on their differences and similarities (Section 3)

- Describing the generalized modelling framework (Section 4)
- Definition of alternative hypotheses using the generalized modelling framework (Section 5)
- Outlining the course of action for completion of a draft decision analysis for spring/summer chinook by October 1997 (section 6)

## 2.0 Description of Models

There are at least 10 models that could potentially be used as prospective models (not all of these have necessarily been suggested for this purpose; Models 2-5 are included because they are similar to others that have been suggested). All of these models are generalized Ricker models, which provides a useful framework for prospective modeling. This framework is modular in the sense that specific hypotheses affect specific terms in the model, allowing different hypotheses to be combined fairly easily. These models are shown in full in Appendix A. A brief description of each follows; complete descriptions are provided in the indicated papers.

### Models 1-5 - MLE models (Deriso et al. 1996)

Models 1-5 are variations on the MLE analysis in Ch. 5. The basic model is a generalized Ricker stock-recruit model, with terms for river mortality (rm; incremental mortality over all life stages to mouth of Columbia River associated with juvenile passage from LGR to Bonneville Dam) and basin-wide year-effects (delta) experienced by all stocks.

1. Base case MLE model, with rm = XN +mu
2. Like 1, except river mortality = passage model estimates * some proportionality constant + mu
3. Like 2, except mu is excluded from river mortality
4. Like 3, except river mortality = passage model estimates (no proportionality constant)
5. Like 4, except year-effects are region-specific (i.e. year effects experienced by Snake River stocks are different from those experienced by Lower-Columbia stocks)

The MLE analysis provides a quantitative basis for evaluating the relative fit of these models to the spawner-recruit data through calculation of AIC and BIC scores. These scores are shown in Table 1 (from Table 5-4 in Deriso et al.; lower scores are better).

**Table 1.**

| Model | AIC (range) | BIC (range) |
|---|---|---|
| 1. | 800.7 | 1145.2 |
| 2. | 799.2 - 805.0 | 1143.7 - 1149.6 |
| 3. | 854.0 - 896.4 | 1114.4 - 1156.8 |
| 4. | 884.3 - 949.1 | 1140.7 - 1205.5 |
| 5. | 862.6 - 890.5 | 1254.5 - 1282.4 |

**Model 6 - Bayesian Prospective Model (Deriso 1997a)**
This model is based on Model 1 above, with an additional term to allow for depensatory mortality at low spawner abundance.

**Model 7 and 8 (Deriso 1997b)**
These models were suggested by Rick Deriso as alternatives to models 9 and 10 below. Model 7 is like Model 4 above, but adds an extra mortality factor (the difference between passage model estimates of passage mortality and the MLE estimate of river mortality). Model 8 is like Model 7, but here the extra mortality factor is not constrained by MLE estimates of river mortality, although the extra mortality is only applied to Snake River stocks for brood years 1972-1990.

**Model 9 - Alpha model (Anderson and Hinrichsen 1997)**
Also a generalized Ricker model. Passage model estimates are used to represent juvenile mortality of juveniles during their passage through or around the juvenile migratory corridor, with an additional region-specific "alpha" term to represent mortality due to regional climatic or ocean conditions and mortality due to indirect effects of passage through or around the hydrosystem.

**Model 10 - Climatic model (Paulsen et al. 1997)**
Generalized Ricker model. Mortality through the migration corridor is proportional to the number of dams encountered. Regional effects of climate, land-use, and habitat conditions are represented explicitly.

## 3.0 Common Components of the Models

Although these models differ in their specific terms, there are 3 major components that these models share. Representation of these components is summarized in **Table 2**.

### 3.1 Intrinsic productivity
All models include Ricker "a" and "b" parameters. The values of these will differ among hypotheses depending on how mortality is allocated to other components of the model. For example, analyses by Rick show that the value of Ricker "a" estimated using the Alpha model (Model 9) is 1.117 lower than the value estimated with Model 1 because the average river mortality estimated by Model 1 was 1.117 higher than the passage mortality estimated by CRiSP in Model 9. The alpha model requires lower intrinsic productivity than MLE Model 1 because passage and delayed mortality are both lower (i.e., less productivity required to overcome less mortality). Ricker "b" values were essentially identical between the two models. Note that this difference in intrinsic productivities would be smaller if FLUSH were used in model 9.

<u>Relevant Hypotheses</u>:

Changes in intrinsic productivity have been mostly associated with habitat and hatchery actions. The approaches discussed for these H actions (e.g. using BayVam model to translate changes in life-stage specific survival rates to overall productivity estimates) are intended to be represented as changes in “a” and “b”.

### 3.2 Direct passage mortality

There are two approaches to modeling direct mortality in the above models. (1) use passage model estimates of in-river survival for non-transported fish, and barge survival for transported fish (models 2-5, 7, 8, 9); or (2) estimate mortality per dam (models 1, 6, and 10). In Ch. 5 comparisons, models that used passage model estimates directly scored slightly worse than the base case according to the AIC criterion, but had similar BIC scores. Note that both of these criteria take the number of MLE-estimated parameters into account (i.e. models with more parameters score worse than models with few parameters). Therefore scores for models that used passage model estimates are probably over-estimated (models actually score worse than indicated) because these estimates were treated as input data, rather than as a set of unknown parameters. Scores were better when passage model estimates were scaled so that the average of the passage model estimates was more similar to the average river mortality. This corrected for consistent differences in passage model estimates relative to the MLE river mortality.

<u>Relevant Hypotheses:</u>

- Any hypothesis about the magnitude and source of mortality that occurs within the juvenile migratory corridor.
- Hypotheses about the effects of proposed management actions (e.g. surface collectors) on overall passage survival.

### 3.3 Extra/Additional/Delayed/Incremental/Differential mortality

This has been given different names, not all of which are accepted by everybody. The acronym “DEAD” encompasses most of the above terms. To avoid making this memo too somber, however, we use the term “extra” mortality, and define it as mortality that occurs outside of the juvenile migration corridor that is not captured in other model components. There are at least two hypothesized sources of this extra mortality.

a) Hydro-related extra mortality - mortality associated with passage through or around the hydrosystem that is incurred in addition to direct mortality of dam/reservoir/barge passage. Hydro-related extra mortality is expressed below Bonneville dam (i.e. in the estuary or ocean), while direct hydro mortality is expressed in the migratory corridor and is captured in passage model mortality estimates. Hypotheses about hydro-related extra mortality are captured in the “mu” term in the MLE-related models, and in the “alpha” term in the Alpha model. The Climatic model also includes this indirectly through the Flow index, which has positive coefficients.

Mu, "alpha", and the Climatic indices are region-specific, allowing for differences in these post-BON responses between Snake River, lower Columbia, and (Alpha model) mid-Columbia stocks. Regional differences are hypothesized to result from differences in the number of dams and reservoirs stocks must pass through or around to reach the ocean, or differences in the proportion of fish transported.

b) Environmental - extra mortality is caused by environmental factors outside of the juvenile migratory corridor. In the MLE models, this extra mortality is captured in the common year effects (the "delta" term). Although these year effects are generally structured so that they affect all stocks equally, some variants of the MLE model (Model 5) were structured to allow for region-specific year effects. These models scored about the same as models with basin-wide "delta" using the AIC criterion but considerably worse using the BIC criterion. In the Alpha model, regional environmental factors are captured in the "alpha" term.

The Climatic model represents environmental factors explicitly, with specific coefficients for climatic indices such as a drought index, date of fall and spring transition, sea surface temperature, and flows. These climatic factors can be stock-specific, region-specific, or basin-wide.

Relevant Hypotheses:

- transportation hypotheses H1-H5 as identified at the Kah-Nee-Ta workshop.
- Early arrival at the estuary of transported fish relative to the spring transition (extra mortality term = f(WTT or FTT).
- Anderson and Hinrichsen (1997) provide an example of a formulation of "alpha" to represent a hypothesis for additional mortality associated with transportation.
- extra mortality associated with changing ocean conditions, exacerbated by ecological/genetic upstream/downstream differences
- no extra mortality

## 4.0 Generalized modeling framework:

The approach taken to integrate the possible prospective models into a single framework considers the framework as a hierarchical structure. At the top level of this structure is a very general linear Ricker model:

(4-1) $$\ln(R_{y,i}) = (1\text{-}p)\ \ln(S_{y,i}) + a_i - b_i S_{y,i} - M_{y,i} - D_{y,j} + \varepsilon_{y,i}$$

where $y$ indexes the year, $i$ indexes individual index stocks, and $j$ indexes regions (Snake River, mid-Columbia, lower Columbia), and $D_{y,j}$ is a general "extra mortality" term that represents any mortality occurring outside of the juvenile migration corridor that is not captured in the other terms. $M_{y,i}$ considers both in-river and transported fish mortality, as estimated by passage models:

$M_{y,i}$ = average direct passage mortality over season, expressed as instantaneous mortality
= -ln(Nb/No), where
Nb = total number of fish alive below Bonneville dam during a season
No = total number of fish reaching top of first reservoir in a season

The extra mortality term ($D_{y,j}$) is defined more precisely at the second level of this hierarchical structure. Through discussions at the June 26-27 prospective modelling meeting and subsequent conference calls, the prospective modelling workgroup has identified two alternative definitions of $D_{y,j}$. One definition is based on the Alpha model (model 9; described in more detail in the enclosed paper "Prospective analysis for the alpha model" by Anderson and Hinrichsen), and defines extra mortality as:

(4-2) $$D_{y,j} = \alpha_{y,j}$$

where $\alpha_{y,j}$ is a region-specific extra mortality term.

The other definition is based on model 7 (described in more detail in the enclosed paper "Draft proposed hypothesis, rationale, and general framework for prospective modelling" by Wilson et al.) and defines extra mortality as:

(4-3) $$D_{y,j} = \Delta m_{y,i} + \delta_y$$

where $\Delta m_{y,i}$ is a stock-specific extra mortality term and $\delta_y$ is a common year effect on mortality of all upstream and downstream stocks.

These two model forms can be shown to be algebraically equivalent (see Appendix B). However, because of differences in their constraints (i.e., the definition of $D_{y,j}$ in equation 4-1), intrinsic productivity using model 9 is consistently lower than that estimated using model 7 (i.e., equation 4-3), with the difference = average $\Delta m$ over brood years 1952-1990 (see Appendix B).

The next level in the hierarchy describes more detailed formulations for $\alpha_{y,j}$ and $\Delta m_{y,i}$. These formulations are described in the other papers in this package.

The major feature of this hierarchical structure is that the top level provides a relatively simple framework to explain to decision-makers, while allowing detailed hypotheses to be formulated at the lower levels. In addition, the general features of alternative *aggregate* hypotheses (i.e., a chain of hypotheses resulting in a particular set of values for the Ricker *a*, Ricker *b*, *M*, and *D* terms) can be compared directly to examine the implications of alternative hypotheses.

Note that the same alternative hypothesis could be modelled using both definitions of $D_{y,j}$ (i.e. 4-2 and 4-3). Although in general we should avoid duplicating all hypotheses in both model forms to avoid redundant analyses, in some cases it might be interesting to see whether the two different formulations of the same hypothesis results in the same decision; if it does not, the differences in the formulations and their underlying assumptions will have to be examined more carefully.

## 5.0 Definition of alternative hypotheses using the generalized modelling framework

Alternative hypotheses about key uncertainties in the life cycle of spring/summer chinook salmon will be expressed as different values of the parameters at different levels in the hierarchical prospective model. For example, assumptions about various components of juvenile passage mortality (e.g. estimates of FGEs, dam passage survival, effects of predator removal programs) will be expressed as different values of $M$ produced by the passage models. Similarly, different representations of extra mortality (either $\alpha_{y,j}$ or $\Delta m_{y,i}$) will reflect alternative hypotheses about the relative importance of hydro and environmental effects in determining extra mortality and differences in extra mortality experienced by upstream and downstream stocks. Note that common year effects can be represented in both model forms, either through the delta term in model 7, or in the Alpha model by expressing the common climate effect as the extra mortality of lower-river stocks (Hypothesis E1 in the Anderson and Hinrichsen paper).

As a first cut at defining alternative hypotheses for extra mortality, the prospective group has decided to focus for now on defining only two hypotheses. One hypothesis attributes extra mortality incurred by upstream stocks to both common year effects and passage through or around the hydrosystem; this hypothesis is operationalized through a framework similar to Deriso et al. (1996). The other hypothesis postulates that extra mortality is largely due to environmental effects, some beyond human influence (e.g. climate) and some partly affected by hydrosystem actions (e.g. river flow and timing of fish arrival in the estuary). Precise definition of the hydrosystem hypothesis is provided in the enclosed paper by Wilson et al., while further description of the environmental hypothesis is provided by Anderson and Hinrichsen. It is important to note that these two hypotheses represent two extreme views; other intermediate hypotheses are possible and will be evaluated in the future. Limiting the analysis to only the two extreme hypotheses is merely a practical way to work out the mechanics of conducting the prospective/decision analysis.

## 6.0 Suggested Approach

The following plan is a possible approach to moving forward with the decision analysis:

1. Agree on a generalized modeling framework (completed). This was the focus of the Prospective modeling group meeting June 26-27 and subsequent conference calls.

2. Generate hypotheses (ongoing). Once the modeling framework has been established, PATH members should go ahead and propose aggregate hypotheses, which include specific hypotheses about each component. These hypotheses should be stated in terms

that are consistent with the generalized framework (e.g. parameter values), along with the ecological justification for the hypothesis (some examples of aggregate hypotheses are shown in **Table 3**). We could also include some habitat or hatchery hypotheses here as well (expressed in terms of Ricker "a" and "b" values) if they were ready.

3. Clearly define the initial set of alternative hypotheses to include in preliminary decision analysis framework (i.e., hydrosystem vs. environment related extra mortality; August 1).

4. Run the initial hypotheses through the generalized modelling framework and generate outcomes (August 29). The output of this might be a table that lists all of the hypotheses, the corresponding performance measures, and the resulting decision choice (preferred management action) if that hypothesis were true (could have multiple choices here using multiple sets of criteria):

| | Performance Measures | | | | | Decision | Decision |
|---|---|---|---|---|---|---|---|
| Hypothesis | S24 | S100 | R24 | R48 | Harvest | (Criterion 1) | (Criterion 2) |
| | - by stock | | | | | | |
| | - Summary: | | Weakest stock | | | | |
| | | | Median stock | | | | |
| | | | Range across stocks | | | | |
| | | | Total harvest across stocks | | | | |

5. Follow-up analyses (to occur after the preliminary decision analysis is completed in October):

a) Generate and define additional alternative hypotheses for extra mortality and other uncertain processes. There should be some coarse filtering of hypotheses here so that those that have no biological basis, or cannot be implemented in a prospective form (e.g. not feasible from historical information to generate a simulated, covarying set of model inputs), are excluded (see Evidence column in **Table 3**). The reason for doing a coarse filtering at this stage is that some of these hypotheses will lead to the same conclusions. Additional filtering can occur once we know which hypotheses are crucial in determining the ranking of management actions.

This coarse filtering stage will probably rely mostly on expert consensus. One could compare alternative models to the stock-recruit data to filter out hypotheses (Approach A in the hydro decision tree paper), but Rick Deriso and Randall Peterman have cautioned that these data are not truly independent data sources since hypotheses may be formulated with the stock-recruitment data in mind. Therefore, different hypotheses may all fit the data equally well since overall survival is the same; only the allocation of that survival to different components differs among hypotheses. Still, the relative likelihoods, AIC, and BIC comparisons could be used as a guide for excluding hypotheses that obviously don't explain the variation in the S-R data or that are over-parameterized.

b) Run the remaining additional alternative hypotheses through the generalized modelling framework and generate outcomes as in Step 4. This will show how important different hypotheses are in determining the decision, which forms the basis for the next round of filtering of hypotheses. The subset of hypotheses that really matter then becomes the focus for:

i) Determining probabilities. Initial filtering of hypotheses will reduce the number of hypotheses to which probabilities are assigned. We need to consider a number of factors in assigning probabilities (likelihoods, other evidence), and then do sensitivity analyses of the decision ranking to the assigned probabilities.

ii) Designing adaptive management actions/research/monitoring plans. The subset of hypotheses that affect the decisions are, by definition, critical uncertainties. Since the decision depends on resolving these uncertainties, these are the hypotheses that need to be tested. Identifying required vs. existing information to test these hypotheses (e.g. **Table 3**) will be helpful.

**References**

ANCOOR. 1993. A comparison of several analytical models used to evaluate management strategies for Columbia River salmon.

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Deriso, R. et al. 1996. Retrospective analysis of passage mortality of spring chinook of the Columbia River. Ch. 5 in Marmorek et al. 1996. PATH Final report on retrospective analyses for fiscal year 1996.

Deriso, R. 1997b. Comments on memo by Anderson et al. 1997. May 16 1997.

Deriso, R. 1997a. Prospective Analysis of spring chinook of the Snake River basin. April 1997.

Paulsen et al. 1997. Chapter 4 Update: Effects of Climate and Land Use on Index Stock Recruitment (draft circulated for internal PATH review). May 30 1997.

Wilson et al. 1997. Draft proposed general framework for prospective modeling with detailed examples for one hypothesis about delayed mortality. August 1, 1997

**Table 2.**

| | | | Extra mortality | |
|---|---|---|---|---|
| **Model** | **Productivity** | **Direct Passage mortality** | **Hydro-related** | **Environmental** |
| 1 (MLE base case) | Ricker a, b | X dam mortality | mu | delta (common) |
| 2 (MLE variant) | Ricker a, b | passage models (scaled with proportionality constant) | mu | delta (common) |
| 3 (MLE variant) | Ricker a, b | passage models (scaled with proportionality constant) | none | delta (common) |
| 4 (MLE variant) | Ricker a, b | passage models | none | delta (common) |
| 5 (MLE variant) | Ricker a, b | passage models | none | delta (regional) |
| 6 BSM prospective model | Ricker a, b | X dam mortality | mu | delta (common) |
| 7 Deriso proposal 1 | Ricker a, b | passage models | delta m (derived from MLE survival) | delta (common) |
| 8 Deriso proposal 2 | Ricker a, b | passage models | “d” for Snake R. stocks, 1972-1990 | delta (common) |
| 9 Alpha model | Ricker a, b | passage models | alpha | alpha |
| 10 Climatic model | Ricker a, b | per-dam mortality | flow | environmental variables |

**Table 3.**

| **Aggregate Hypothesis (examples only)** | **Model parameter values** | | | | | **Examples of evidence to assess validity of hypothesis** | |
|---|---|---|---|---|---|---|---|
| | **p** | **a** | **b** | **M** | **y** | **Existing** | **Required** |
| A1. Extra post-BON mortality is related to hydrosystem direct mortality; common year effects | | | | | $=\Delta m+\delta$<br>$\Delta m=f(M, m, WTT)$ | posterior probability of that combination of model parameters Wilson et al. (attached) | |
| A2. Extra mortality is unrelated to the hydrosystem, is purely a function of ocean conditions, and varies with stock's region of origin; no common year effects | | | | | $=\alpha$<br>$\alpha=f(ocean)$ | posterior probability of that combination of model parameters<br><br>limited CWT data on ocean distributions (Paulsen 1997)<br><br>compare ocean survival of chinook in CA to ocean survival of chinook in WA, OR | to be filled in |
| A3. Extra mortality is not necessarily proportional to direct mortality (e.g. estuary arrival time relative to spring transition); no common year effects | | | | | | posterior probability of that combination of model parameters<br><br>Hinrichsen et al. 1997 | |

## Appendix A. Summary of Potential Prospective Models

### MLE model (Deriso et al.):

(1) Ch.5 model 1 $\ln(R_{t,i}) = \ln(S_{t,i}) + a_i - b_i S_{t,i} - XN_{t,i} - \mu_{t,j} + \delta_t + \varepsilon_{t,i}$

(2) Ch. 5 models 5-8 $\ln(R_{t,i}) = \ln(S_{t,i}) + a_i - b_i S_{t,i} - (q * M + \mu_{t,j}) + \delta_t + \varepsilon_{t,i}$

(3) Ch. 5 models 9-12 $\ln(R_{t,i}) = \ln(S_{t,i}) + a_i - b_i S_{t,i} - q * M + \delta_t + \varepsilon_{t,i}$

(4) Ch. 5 models 13-16 $\ln(R_{t,i}) = \ln(S_{t,i}) + a_i - b_i S_{t,i} - M + \delta_t + \varepsilon_{t,i}$

(5) Ch. 5 models 25-28 $\ln(R_{t,i}) = \ln(S_{t,i}) + a_i - b_i S_{t,i} - M + \delta_{t,j} + \varepsilon_{t,i}$

where:
$q$ = proportionality constant (estimated)
$M_{t,i}$ = ln (passage survival estimated by CRiSP or FLUSH)
$N_{t,i}$ = # of dams passed by stock $i$ in year $t$
$X$ = per-dam mortality for X-type dams
$\mu_{t,j}$ = incremental mortality incurred by Snake River stocks to the mouth of the Columbia
$\delta_t$ = common year-effects experienced by all stocks

(6) **BSM prospective model (Deriso):**

$$\ln(R_{t,i}) = (1+p)\ln(S_{t,i}) + a_i - \beta_i S_{t,i} + \ln(\beta) - XN_{t,i} - \mu_{t,j} + \delta_t + \varepsilon_{t,i}$$

with sensitivity to a modification for additional depensation: $R/S = R * (S/S_{min})^d$ for $S < S_{min}$

where:
$p$, $d$ = depensation parameters
$S_{min}$ = minimum observed number of spawners

**Proposed by Deriso in his May 16 comments to memo by Anderson et al.:**

(7)

$$\ln(R_{t,i}) = \ln(S_{t,i}) + a_i - b_i S_{t,i} - M - \Delta m + \delta_t + \varepsilon_{t,i}$$

where:
M = ln(passage survival estimate from CRiSP or FLUSH)
m = river mortality estimated by MLE model (1) above - M

(8)

$$\ln(R_{t,i}) = \ln(S_{t,i}) + a_i - b_i S_{t,i} - M - d + \delta_t + \varepsilon_{t,i}$$

where:
d = “delay mortality” parameter for Snake River stocks, brood years 1972-1990

(9) **Alpha model (Anderson and Hinrichsen):**

$$\ln(R_{t,i}) = (1+p)\ln(S_{t,i}) + a_i - b_i S_{t,i} + \ln(crisp_{t,i}) + \alpha_{t,j} + \varepsilon_{t,i}$$

where:
α = a region-specific additional mortality term

(10) **Climatic model (Paulsen and Hinrichsen):**

ln R($t$,$i$) = ln S($t$,$i$) + a($i$) - b($i$)S($t$,$i$) + c($1$)X($1$,$t$,$i$) + … + c($n$)X($n$,$t$,$i$) + d($1$,$r$)Z($1$,$r$,$t$) +… + d($n$,$r$)Z($n$,$r$,$t$) + e($1$)U($1$,$t$) + … + e($n$)U($n$,$t$) + $\varepsilon$ ($t$,$i$)

where:

*t* indexes brood year;
*i* indexes stock;
*r* indexes region (Lower Columbia, Snake, and mid-Columbia);
c($1$) … c($n$) are the estimated effects of stock-specific environmental factors X($1$,$t$) .. X($n$,$t$) (including the number of mainstem dams encountered during outmigration);
d($1$) … d($n$) are the estimated effects of regional environmental factors Z($1$,$t$) .. Z($n$,$t$);
e($1$) … e($n$) are the estimated effects of basin-wide environmental factors U($1$,$t$) … U($n$,$t$);
ε($t$,$i$) = error term, assumed to be distributed N (0, $^2$), IID.

## Appendix B.
## Some thoughts about
## S-R model equivalence

July 9, 1997

R. Deriso

Recall from previous documents that we have a model proposed by Deriso and others:

$$\ln(R_{t,i}) = (1+p)\ln(S_{t,i}) + a_i - b_i S_{t,i} - M_{t,i} - \Delta m_{t,i} + \delta_t + \varepsilon_{t,i} \qquad \text{(a)}$$

and a model proposed by Anderson and others:

$$\ln(R_{t,i}) = (1+p)\ln(S_{t,i}) + a_i' - b_i' S_{t,i} - M_{t,i} + \alpha_{t,j} + \varepsilon_{t,i} \qquad \text{(b)}$$

The two models have different Ricker "a" coefficients because constraints on the other parameters differ.

In (a), there is the constraint that

$$\sum_{t=1952}^{1990} \delta_t = 0\,.$$

In (b), there are the constraints that

$$\sum_{t=1952}^{1990} \alpha_{t,1} = 0 \quad \text{and} \quad \sum_{t=1952}^{1990} \alpha_{t,2} = 0 \quad \text{and} \quad \sum_{t=1952}^{1990} \alpha_{t,3} = 0$$

We can write model (b) in the terms of model (a) parameters by adding and subtracting a scalar. Let

$$\alpha_{t,j} = -\Delta m_{t,i} + \delta_t - \frac{1}{39}\left\{ \sum_{k=1952}^{1990} -\Delta m_{k,i} + \delta_k \right\}$$

and let the Ricker "a" parameter in model (b),

$$a_i' = a_i + \frac{1}{39}\left\{ \sum_{k=1952}^{1990} -\Delta m_{k,i} + \delta_k \right\}.$$

So far, results have the Ricker "b" parameter estimates as equal between model (a) and (b).

One problem with the alpha model is that it has lots of alpha's to estimate – far more than AIC or BIC would permit. One solution is to seek a flexible approximation for the alpha's. Such an approximation is already used in model (a) where the approximation is made that

$$\Delta m_{t,i} = n_{t,i} X \ + \mu_{t,i} - M_{t,i}$$

We could reach model equivalence if the same approximation was used in model (b) for $\Delta m_{t,i}$. Results of AIC, BIC along with comparisons between adjusted alphas and adjusted mu's all support the use of such an approximation. [For those of you who haven't seen the model comparison memo written by Anderson and myself, I've attached the part of it I wrote as Appendix B-I].

### Appendix B-I : email excerpt of May 13,1997

I'll write Jim Anderson's (JA) model as: lnR= lnS +a' -b'S +alpha -crispM
I'll write MLE model 1 as : lnR= lnS +a -b S +delta -rm

1st result: intrinsic productivity in (JA) model is lower than in MLE model. Note that average (alpha) over all years of data = average delta = 0; therefore if b=b' (which is nearly exact) then average over all years of data to get

a' = a +average(crispM) - average(rm)

average(crispM) = 0.554 for Snake R stocks (1952-1990 byr)
average(rm) = 1.671 for Snake R stocks (1952-1990 byr)

therefore, a' = a -1.117

2nd result: correlation between adjusted alpha and adjusted MU is 0.997 and the adjusted
alpha is offset by 1.117 relative to adjusted MU.

As seen in the model structure above the alpha needs to be adjusted by crispM and the MU needs to be adjusted by delta - first/level/dam effects to get comparability.

Define A = alpha -crispM
Define B = delta -rm

then

A = intercept +slope*B where
A = 1.117 +1.0 *B where
SUMMARY OUTPUT

Regression Statistics

| | |
|---|---|
| Multiple R | 0.997528614 |
| R Square | 0.995063335 |
| Adjusted R Square | 0.994929912 |
| Standard Error | 0.08009312 |
| Observations | 39 |

ANOVA

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| Regression | 1 | 47.84198979 | 47.84198979 | 7457.938713 | 2.78257E-44 |
| Residual | 37 | 0.237351591 | 0.006414908 | | |
| Total | 38 | 48.07934138 | | | |

| | Coefficients | Standard Error | t Stat | P-value | Lower 95% | Upper 95% |
|---|---|---|---|---|---|---|
| Intercept | 1.117705835 | 0.023222872 | 48.12952664 | 5.66701E-35 | 1.070651873 | 1.164759797 |
| X Variable 1 | 1.000612599 | 0.011586615 | 86.35935799 | 2.78257E-44 | 0.977135911 | 1.024089287 |

Conclusion: Offset of 1.117 in alpha is recovered by offset of 1.117 in Ricker "a" parameter. Reduction in correlation from 1.0 to 0.997 is due to use of first/level/dam parameterization for 1952-1969 byrs.

In the BSM, parameterization of JA model would be accomplished by replacing MLE model 1 with JA model; i.e. replace a+delta-rm with a'+alpha-crispM for each projected year; some caution should be taken to correctly project JA's hypothesis about autoregressive properties of alpha (as opposed to current BSM which specifies autoregressive properties for delta).